Q.P. Code - 22437

Fourth Semester B.Sc. Degree Examination, September 2020

(Semester Scheme)

MATHEMATICS

Paper IV

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. If H is a normal subgroup of a finite order, then prove that $O\left(\frac{G}{H}\right)=\frac{O(G)}{O(H)}$.

2. Prove that $H=\{1,-1\}$ is a normal subgroup of $G=\{1,-1,i,-i\}$ under multiplication.

3. If $f:G\to G'$ is a homomorphism then prove that $f\left(a^{-1}\right)=[f(a)]^{-1}, \forall\ a\in G$.

4. Define homomorphism and isomorphism of Groups.

5. Let $f:G\to G'$ be an isomorphism. If G is abelian then G' us also abelian.

6. Expand $e^x \sin y$ by Taylor's theorem in term of x and y upto second degree terms.

7. Find the critical point of $x^3y^2(1-x-y)$.

8. Prove that $\beta(m,n)=\beta(n,m)$.

9. Compute $\Gamma\left(-\frac{7}{2}\right)$.

10. Evaluate : $\int_0^1 x^{\frac{5}{2}}(1-x)^4\,dx$.

11. Evaluate $\int_0^\infty x^7 e^{-x^2}\,dx$.

12. Solve : $(D^2 + 5D + 6)y = 0$.

13. Find the particular integral of $(D^2 + 2D + 1)y = 2e^{2x}$.

14. Verify the exactness of $x^2(1+x)y'' + 2x(2+3x)y' + 2(1+3x)y = 0$.

15. Verify the conditions for integrability for
$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$.

16. Evaluate : $L\left[e^{-4t} + 3e^{-2t}\right]$.

17. Evaluate : $L^{-1}\left[\frac{1}{(S-4)^3}\right]$.

18. Find the Laplace transform of $\left\{\frac{\sin t}{t}\right\}$.

19. Define feasible solution of LPP.

20. Draw the graph of the solution of $x + 2y \geq 2$, $3x + y \geq 3$ and $x > 0, y > 0$.

II. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H$, $\forall\, g \in G$.

2. Show that every factor Group of a cyclic group is cyclic.

3. Prove that $f : G \to G'$ be a homomorphism from the Group G into G' with Kernel K. Then K is a normal subgroup of G.

4. State and prove Cayley's Theorem.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the first six terms of the expansion of the function $e^x \log(1+y)$, in a Taylor's series in the neighbourhood of the point (0, 0).

2. Test for maximum and minimum of the function $f(x, y) = x^3 + y^3 - 3xy$.

3. Show that a rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

4. Evaluate $\dfrac{\Gamma(3)\,\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{9}{2}\right)}$

5. Show that $\int_0^1 \dfrac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = \beta(m, n)$.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Solve : $(D^2 - 2D + 5)y = \sin 3x$.

2. Solve : $(D^2 + 4)y = \sin^2 x$.

3. Solve : $(2x-1)^3 \dfrac{d^3y}{dx^3} + (2x-1)\dfrac{dy}{dx} - 2y = 0$.

4. Solve by changing the dependent variable :

$\dfrac{d^2y}{dx^2} - 2\tan x\dfrac{dy}{dx} - (a^2+1)y = e^x \sec x$, by reducing it to normal form.

5. Solve : $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Evaluate

(a) $L(1+t)^3$

(b) $L(t^3 + 3t^2 - 6t + 8)$

2. Find $L^{-1}\left[\dfrac{3S+2}{2S^2 - 4S + 3}\right]$.

3. Solve $9y'' - 6y' + y = 0$ given $y(0) = 3$ and $y'(0) = 1$.

VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Show that the set $S = \{(x_1, x_2)/2x_1 + 3x_2 = 7\}$ is a convex set in $\mathbf{R}^2$.

2. Find the maximum value of $z = 2x + 3y$, subject to the constraints $x + y \le 30$, $x - y \ge 0$, $y \ge 3$, $0 \le x \le 20$, $0 \le y \le 12$.

3. Use simplex method to solve the following LPP.

   Maximize : $z = x - y + 3z$,

   Subject to the constraints :

$$x + y + z \le 10$$
$$2x - z \le 2$$
$$2x - 2y + 3z \le 0$$
$$x, y, z \ge 0$$