Q.P. Code – 42439

Fourth Semester B.Sc. Degree Examination, September 2020

(CBCS Scheme)

Mathematics

Paper 4.1 – ALGEBRA AND CALCULUS – II

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

PART – A

I. Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Evaluate : $\int_C [(2y + x^2)dx + (3x - y)dy]$ along the curve $x = 2t$ and $y = t^2 + 3$ where $0 \le t \le 1$.

2. Evaluate : $\int_1^2 \int_0^3 (x^2 + 3xy)dx\,dy$.

3. If $\vec{f} = x^2yi - 2xzj + 2zyk$, find div $\vec{f}$ at (1, 2, 3).

4. Show that $\vec{f} = (\sin y + z)i + (x\cos y - z)j + (x - y)k$ is irrotational.

5. Find the order of the elements of the multiplicative group $G = \{1, w, w^2\}$ of the cube roots of unity.

6. Find all the distinct right cosets of $H = \{0, 4, 8\}$ in $(z_{12}, +_{12})$.

7. Prove that intersection of two normal subgroups of a group G is also a normal subgroup.

8. Define Kernel of Homomorphism.

PART – B

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

9. Evaluate : $\iint_R \frac{dx\,dy}{x + y + 1}$ over the square $R : 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le x, y \le 1$.

10. Evaluate : $\int_1^2\int_1^2\int_1^2\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)dx\,dy\,dz$.

11. Find the directional derivative of the function $\phi(x, y, z) = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of $2\hat{i} + \hat{j} + 2\hat{k}$.

12. If $\phi = x^2y^3z^4$ and $f = xy + yz + zx$ find $\nabla(\phi f)$.

13. If H and K are any two subgroups of a group 'G' then HK is a subgroup of G if and only if $HK = KH$.

14. Find the number of generators of the cyclic group $(z_{18}, +_{18})$, write all the generators.

15. If H is the only subgroup of finite order m in the group G then show that H is normal in G.

16. Let $f : G \to G'$ be an isomorphism, if G is abelian then show that G' is also abelian.

PART – C

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

17. Evaluate : $\int_C[(x^2 - 2xy)dx + (x^2y + 3)dy]$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

18. Evaluate : $\int_0^a\int_0^{\sqrt{a^2-x^2}}\sqrt{a^2 - x^2 - y^2}\,dy\,dx$ by changing the order of integration.

19. Evaluate : $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2 + y^2)dy\,dx$ by changing into polar co-ordinates.

20. Find the volume of common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

21. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{4} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

22. If ϕ is a scalar function and $\vec{f}$ is a vector function then prove that $\text{div}(\phi \vec{f}) = \phi \text{ div } \vec{f} + \vec{f} \cdot \text{grad}\,\phi$.

23. State and prove Green's theorem.

24. Verify Stoke's theorem for the function $\vec{F} = y^2 \hat{i} + xy \hat{j} - xz \hat{k}$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

V. Answer any **THREE** of the following : **(3 × 5 = 15)**

25. If 'a' and 'x' are any two elements of a group G then prove that $o(a) = o(xax^{-1})$.

26. Prove that every subgroup of a cyclic group is cyclic.

27. If H is a subgroup of a group G, then for all $a \in G$ prove that $[a] = Ha$.

28. State and prove Fermat's theorem.

VI. Answer any **THREE** of the following : **(3 × 5 = 15)**

29. Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H$, $\forall g \in G$.

30. If N is a normal subgroup of G and H is any subgroup G then prove that NH is a subgroup of g.

31. State and prove Fundamental theorem of Homomorphism.

32. State and prove Cayley's theorem.
